SOME TRANZIENT THERMO ELASTIC PROBLEM OF THIN CLAMPED RECTANGULAR PLATE

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Abstract- This paper is concerned with transient Thermoelastic problem in which we need to determine the Temperature distribution and thermal deflection of a thick Clamped rectangular plate when the boundary condition are Known. Integral transform techniques are used to obtain the Solution of the problem.

Keywords: Thick rectangular plate, transient problem , thermoelastic problem, finite Fourier cosine , Transform and Marchi--Zgrablich transform

1 INTRODUCTION

The method proposed here enables us to give a systematic treatment to obtain thermoelastic solutions of clamped rectangular plate with any desired accuracy. Since Nowacki [44] has treated the steady-state thermal stresses in a circular plate subjected to an axisymmetric temperature distribution on the upper face with zero temperature on the lower face and the circular edge respectively. Among them, Roychoudhary [63] discussed the normal deflection of a thin clamped rectangular plate due to ramp type heating of a concentric circular region of the upper face. This satisfies the time – dependent heat conduction equation. On the other hand, Varghese [91] have determined the thermal stresses in a rectangular plate due to internal heat generation within it. In the present chapter, an attempt is made to determine the temperature distribution and thermal deflection at any point of the plate occupying the space *D:* $\{(x, y, z) \in R^3 : 0 \le x \le a, \quad 0 \le y \le b, 0 \le z \le h\}$ with the known boundary conditions. Finite Fourier sine transform and Laplace transform techniques are used to find the solution of the problem. Numerical estimate for the temperature distribution is obtained and depicted graphically.

2 STATEMENT OF THE PROBLEM

Consider a thick isotropic rectangular plate occupying the space *D*. The differential equation satisfied by the deflection

$\omega(x, y, t)$ as Roy Choudhary [63] is

$$D\nabla^4 \omega(x, y, t) = \frac{-\nabla^2 M_T(x, y, t)}{1-\nu} \quad (1)$$

where,

ν is the Poisson's ratio of the plate material ,

M_T Denotes the thermal momentum of the plate and

D denote the flexural rigidity,

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2}$$

The resultant thermal momentum M_T is defined as

$$M_T(x, y, t) = \alpha E \int_0^h z T(x, y, z, t) dz \quad (2)$$

Where α, *E* are the linear coefficient of thermal expansion of the material, and Young's modulus respectively.

Since the edge of the rectangular plate is fixed and clamped,

$$\omega = \frac{\partial^2 \omega}{\partial x^2} = \frac{\partial^2 \omega}{\partial y^2} = 0, \text{ at } x = a \text{ and } y = b$$

The temperature of the plate at time t satisfying the differential equation as Nowacki [44] is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{k}\frac{\partial T}{\partial t}$$

subject to the initial and boundary conditions as

$$T(x, y, z, 0) = 0$$

$$\left[T(x, y, z, t)\right]_{x=0} = 0$$

$$\left[T(x, y, z, t)\right]_{x=a} = 0$$

$$\left[T(x, y, z, t)\right]_{y=0} = 0$$

$$\left[T(x, y, z, t)\right]_{y=b} = 0$$

$$\left[\frac{\partial T(x, y, z, t)}{\partial z}\right]_{z=0} = 0$$

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$$\left[\frac{\partial T(x,y,z,t)}{\partial z}\right]_{z=h} = f(x,y,t)$$

where k is the thermal diffusivity of the material of the plate. Equations (1) to (11) constitute the mathematical formulation of the problem under consideration.

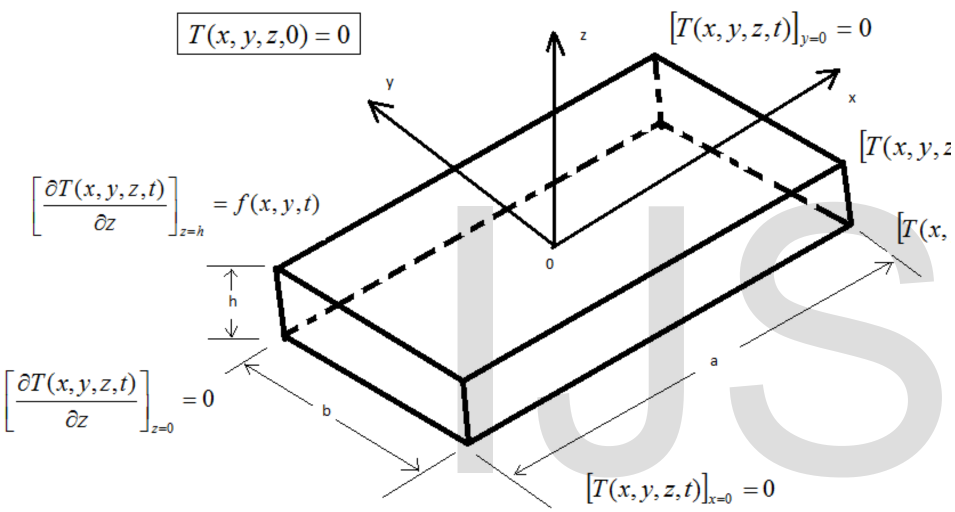

Figure 1: Geometry of the problem

3 SOLUTION OF THE PROBLEM

By applying finite Fourier sine transform w.r.to x and y successively and Laplace transform and further using their inverses to the equations (4) to (11), one obtains the expression for temperature distribution as

(11)

$$T(x,y,z,t) = \frac{8k}{abh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l \cos(\lambda_l z)\,\sin(px)\,\sin(qy)$$

$$\times\int_0^t \overline{\overline{f}}(m,n,t')e^{-k\left[p^2+q^2+\lambda_l{}^2\right](t-t')}dt'$$

Where l, m, n are the positive integers

$$\lambda_l = \frac{l\pi}{h},$$

$$\overline{\overline{f}}(m,n,t) = \int_0^a\int_0^b f(x,y,t)\sin(px)\sin(qy)\,dx\,dy$$

4 DETERMINATION OF THERMAL DEFLECTION

Substituting the value of temperature distribution T(x, y, z, t) from equation (12) in equation (3.2), one obtains

$$M_T(x,y,t) = \frac{8k\alpha E}{abh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^l\left(\int_0^h z\cos(\lambda_l z)dz\right)\sin(px)\,\sin(qy)$$

$$\times\int_0^t \overline{\overline{f}}(m,n,t')e^{-k\left[p^2+q^2+\lambda_l{}^2\right](t-t')}dt'$$

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We assume that the solution of equation (1) satisfying equation (3) as

$$\omega(x,y,t)=\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\omega_{mn}(t)\sin(px)\sin(qy)$$

(14)

Using the equations (13) and (14) in (1), one obtains

$$\omega_{mn}(t)=\frac{8k\alpha E}{D(1-\nu)abh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{l}\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^{2}(p^2+q^2)}\right)$$

$$\times\int_0^t \overline{\overline{f}}(m,n,t')e^{-k\left[p^2+q^2+\lambda_l^2\right](t-t')}dt'$$

(15)

Substituting the value of $\omega_{mn}(t)$ in equation (14), one obtains the expression for thermal deflection as

$$\omega(x,y,t)=\frac{8k\alpha E}{D(1-\nu)abh}\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{l}\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^{2}(p^2+q^2)}\right)$$

$$\times\sin(px)\sin(qy)\int_0^t \overline{\overline{f}}(m,n,t')e^{-k\left[p^2+q^2+\lambda_l^2\right](t-t')}dt'$$

(16)

5 SPECIAL CASE AND NUMERICAL RESULTS

Set $f(x,y,t)=(1-e^{-t})(x^2-ax)(y^2-by)$, $\beta=\dfrac{32k}{abh}$,

$\gamma=\dfrac{16k\alpha E}{D(1-\nu)abh}$, a = 1m, b = 2m, h = 0.2m,

t = 1sec and k= 0.86 in equations (12) and (16), we obtain

$$\frac{T(x,y,z,t)}{\beta}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{l}\left[(-1)^m-1\right]\left[(-1)^n-1\right]\left(\frac{1}{p^3q^3}\right)$$

$$\times\cos(\lambda_l z)\ \sin(px)\ \sin(qy)$$

$$\times\int_0^1(1-e^{-t'})e^{-k\left[p^2+q^2+\lambda_l^2\right](1-t')}dt'$$

$$\frac{\omega(x,y,t)}{\gamma}=\sum_{l=0}^{\infty}\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}(-1)^{l}\left[(-1)^m-1\right]\left[(-1)^n-1\right]$$

$$\times\left(\frac{\lambda_l h\sin(\lambda_l h)+\cos(\lambda_l h)-1}{\lambda_l^{2}p^3q^3(p^2+q^2)}\right)\sin(px)\sin(qy)$$

$$\times\int_0^1(1-e^{-t'})e^{-k\left[p^2+q^2+\lambda_l^2\right](1-t')}dt'$$

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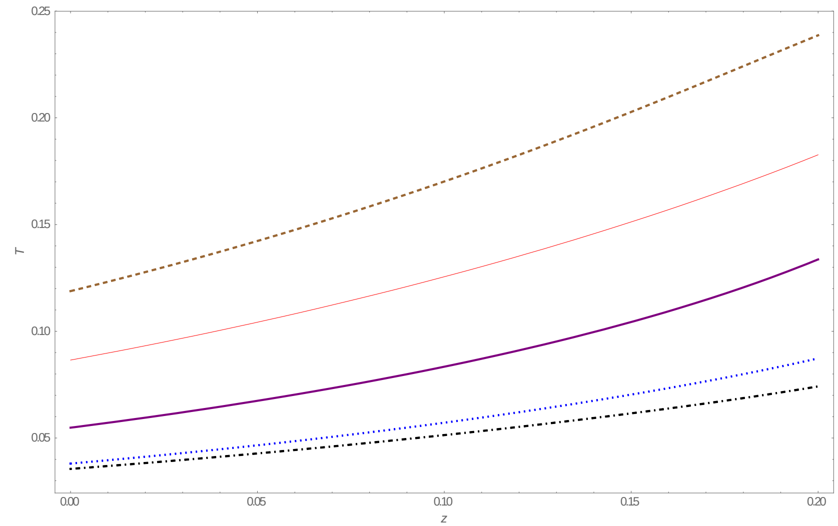
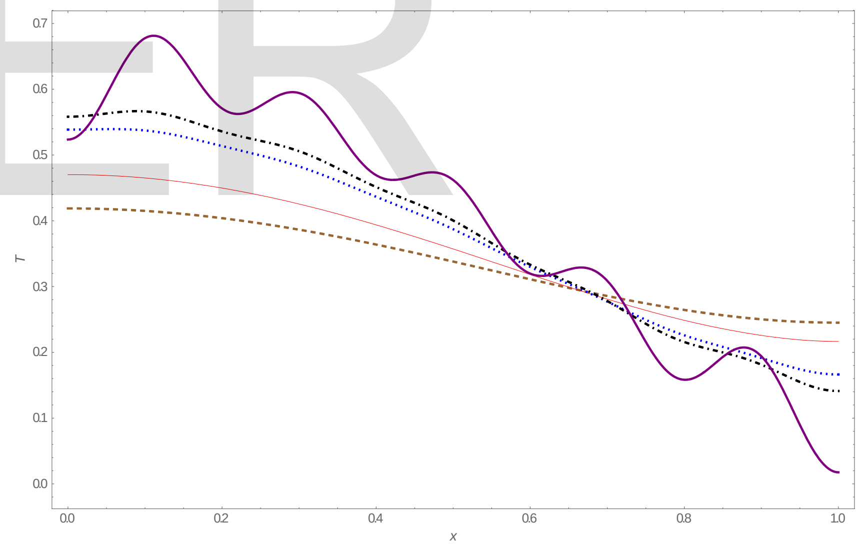
Figure 2 and 3 illustrate the temperature profile and thermally induced deflection. Figure 2 depicts the temperature distribution along x and z axis for different values of time. It was observed that temperature increases along z axis may be due to sectional heat supply during which thermal deformation taken at outer edge. Along x axis due to compressive force the value shows decreasing trend towards outer edge. In Figure 3 thermal deflection happens more towards center and it may be due to tensile force whereas values are attending zero at both inner and outer edge due to compressive force. This satisfies the fixed and clamped boundary condition

6 CONCLUSION

The temperature distribution and thermal deflection of a thin rectangular plate have been obtained, with the aid of finite Fourier sine transform and Laplace transform techniques when the stated boundary conditions are known. The results are obtained in the form of infinite series. The series solutions converge provided we take sufficient number of terms in the series. The expressions are represented graphically. The temperature distribution, and deflection that are obtained can be applied to the design of useful structures or machines in engineering applications.



(a)



(b)

Figure 2: Temperature distribution profile

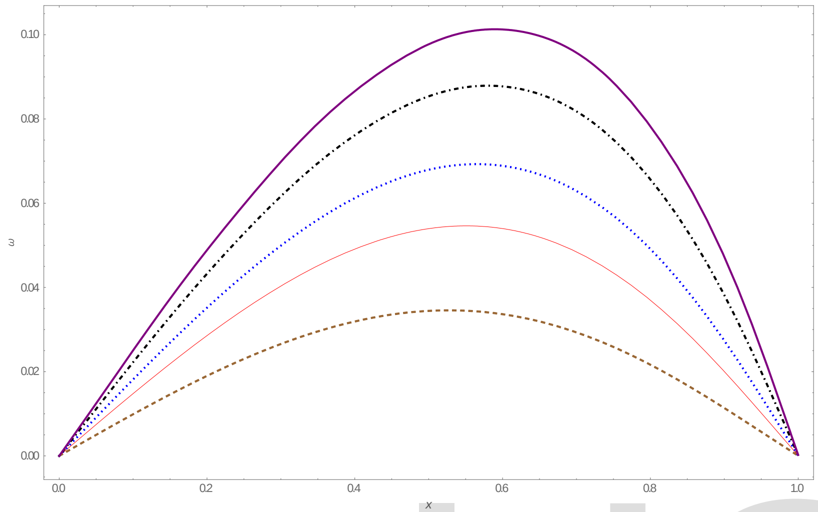


Figure 3: Thermal deflection profile

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